3. A Solution to Roulette

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1. Introduction

Roulette is a well-designed device for gamblers. For statisticians, it is a giant-size puzzle. In this paper, a combined approach of probability, statistics, combinatorics, and cryptography will be used to solve the roulette puzzle. Firstly, I shall prove that the number-arrangement on both American and European roulette wheels are not random; they both have their own unique numerical pattern or numerical order. Completion of this proof means actually that the system is theoretically breakable.

Secondly, there exists a "Red-odd versus black-even" differential. It is known that its red numbers are equal to its black numbers and its odd numbers are equal to its even numbers; so its red-odd numbers should be equal to its red-even numbers and its black-odd numbers should be equal to its black-even numbers on both American and European roulette wheels. But, through observation, they are not equal to each other. This differential can not be eliminated under current 36-number scheme, only a modified 48-number scheme will be able to. The existence of this differential means that the system is not flawless, or it is practically solvable.

Next, a success-region method is suggested. After non-randomness has been proved, we actually converted the sequence of numbers on a roulette wheel into a sequence of natural numbers instead of being a set of random numbers. So, we may choose a segment or any segment of this sequence as our success-region. Besides, during the proving process, we found that in a three-way division, for some areas their probabilities are not equal.

Finally, random walk or Markovian models have to be mentioned due to the existence of zero and double-zero on the roulette wheels. But, my colleagues may soon find out that once the non-randomness had been proved, then the roulette game does not belong to random walk models, and classical gamblers' ruin should not apply. This eventually leads to the conclusion that roulette game is no longer a game of chance.

2. The Solution to American Roulette

At first, let us take a look at the diagram of an American roulette. In this diagram, the two numbers 0 and 00 are colored green. Then, starting with number 1 and proceeds clockwise, red and black are colored in every other space. See Figure 1.

American roulette follows a trichotomy rule or a three-way division in which 356 numbers are equally divided into three sets. Then, a double-looping technique is applied, in which numbers in each set are selected methodically. Such looping techniques are common in modern computer programming although roulette was invented at a much earlier date. It is known that Pascal invented the first calculating machine in 1642. Although the inventor of roulette is remained anonymous, it is possible that Pascal simultaneously invented the game.

The procedure of making an American roulette is as following:

Step 1. Two zeros are placed in the front. 36 numbers are equally divided into three sets as shown in Figure 2.

Step 2. The numbers in each set are now selected methodically: (1) the tiles of numbers 1 and 2 are taken from the first set, (b) the tiles of numbers 13 and 14 from the second set, (c) the tiles of numbers 35 and 36 from the far end of the third set, inversely, and (d) the tiles of numbers 23 and 24 are taken from the far end of the second set, also inversely. The result is shown in Figure 3. (The shadowing parts indicate the direction of each move).

Step 3. The same process is repeated. (a) the tiles of numbers 3 and 4 are taken from the first set, (b) the tiles of numbers 15 and 16 from the second set, (c) the tiles of numbers 33 and 34 from the far end of the third set, inversely, and (d) the tiles of numbers 21 and 22 are taken from the far end of the second set, also inversely.

Step 4. Again, the same process is repeated. (a) the tiles of numbers 5 and 6 are taken from the first set, (b) the tiles of numbers 27 and 18 from the second set, (c) the tiles of numbers 31 and 32 are taken from the far end of the third set, inversely, and (d) the tiles of numbers 19 and 20 are taken from the far end of the second set, also inversely.

Step 5. At this point, all the tiles in the second set have been used up. Consequently, (a) the tiles of numbers 7 and 8 are taken from the first set, (b) the tiles of numbers 11 and 12 from the first set, and (c) the tiles of numbers 29 and 30 from the third set, inversely. (d) the tiles of numbers 25 and 26 are taken from the third set, also inversely.

Step 6. Finally, (a) the tiles of numbers 9 and 10 are taken from the first set, and (b) the tiles of numbers 27 and 28 are taken from the third set, but inversely. All the results, so far, are shown in Figure 4.

In next step, these two rows of numbers are separated and connected from end to end. When it is expanded into a full circle, it is an American roulette wheel. Now, the odd and even numbers naturally appear in every other two spaces. Also it is alternatively in color of red and black in every other single space. Thus, American roulette may not completely preserve the idea of randomness, it does carry the beauty of symmetry to its utmost.

3. The Solution to European Roulette

The diagram of an European roulette is shown in Figure 5. In this diagram, the number 0 is colored green. Next, starting with number 32, the red and black spaces are colored alternatively.

The making of European roulette is a little bit more sophisticated than that of American roulette. It does preserve the idea of randomness better, but does not possess the beauty of symmetry. The procedure of making an European roulette, step by step, is as following:

Step 1. Place the single-zero in the front, then equally divide all other 36 numbers into 4 rows with 9 numbers in each row. The first row includes numbers 1 to 9, the second row includes numbers 10 to 18, the third row includes numbers 19 to 27, and the fourth row includes numbers 28 to 36. We shall randomly select one number from each row. The interesting part is that the row selection is also random.

In the very first round, we randomly select number 7 from the first row, number 16 from the second row, number 25 from the third row, and number 35 from the fourth row. We call these four numbers our insertion group, since we want to put these numbers aside for the time being, and to be inserted later.

Up to now, the framework for an European roulette looks like in Figure 6.

Step 2. In the second round, (a) take number 32 from the fourth row, (b) take the number 15 from the second row, (c) take number 19 from the third row, (d) take number 4 from the first row. Notice that the row selection continues to be random. These four numbers are our first group. The situation now is shown in Figure 7.

Step 3. In the third round, (a) take number 21 from the third row, (b) take number 2 from the first row, (c) take number 17 from the second row, and (d) take number 34 from the fourth row. These four numbers are called our second group.

Step 4. In the fourth round, (a) take number 6 from the first row, (b) take number 27 from the third row, (c) take number 13 from the second row, and (d) take number 36 from the fourth row. These numbers are called our third group.

Step 5. In the fifth round, (a) take number 11 from the second row, (b) take number 30 from the third row, (c) take number 8 from the first row, and (d) take number 23 from the third row. These four numbers are called our fourth group.

Step 6. In the sixth round, (a) take number 10 from the second row, (b) take number 5 from the first row, (c) take number 24 from the third row, and (d) take number 33 from the fourth row. These four numbers are called our fifth group.

Step 7. In the seventh round, (a) take number 1 from the first row, (b) take number 20 from the third row, (c) take number 14 from the second row, and (d) take number 31 from the fourth row. These four numbers are called our sixth group.

Step 8. In the eighth round, (a) take number 9 from the first row, (b) take number 22 from the third row, (c) take number 18 from the second row, and (d) take number 29 from the fourth row. These four numbers are called our seventh group.

Step 9. In the final round, (a) take number 28 from the fourth row, (b) take number 12 from the second row, (c) take number 3 from the first row, and (d) take number 26 from the third row. These four numbers are called our eighth group.

We had completed our selection process, and the current situation on the framework is shown in Figure 8.

Step 10. After the selection process is completed, all groups are in good order. Now we shall expand it into a full circle with the single-zero in the very front. Keep in mind that, up to now, our insertion group is still in outside, no action has been taken yet.

Next, we shall start our insertion process, we shall insert those four numbers into the circle at random. The current situation on the framework is shown in Figure 9.

Step 11. This is our final step of making an European roulette. After the insertion process is completed, we shall color the red and black alternatively on each single space. Then, the European roulette, or the French roulette, is completed. Its diagram had actually been shown before, as previous Figure 5. See Figure 5.

4. "Red-odd versus Black-even" Differential

It is known that, in a roulette, its red numbers are always equal to its black numbers, and its odd numbers are always equal to its even numbers, they are 18 each. But it is far from the general thinking that its red-odd numbers are equal to its red-even numbers, and its black-odd numbers are equal to its black-even numbers; they should be 9 each accordingly. In fact, there are 10 red-odd numbers an 8 red-even numbers; and there are 10 black even numbers and 8 black-odd numbers. This statement is held true for both American and European wheel.s Readers may verify this by checking both American and European roulette layouts as shown in Figure 10 and 11.

We name this differential as "red-odd versus black-even" for sake of convenience. Literally, it should be named as "red-odd versus red-even and black-odd versus black even" differential.

Obviously, this fact shall affect the play. Suppose that a player makes a simple bet on red, black, odd, even, small (numbers 1 to 18), or large (19 to 36). We call these simple bets as our normal play. It follows:

The probability of winning is 18/38 = 0.4737.

The probability of losing is 20/38 = 0.5263 (due to the existence of 2 zeroes).

The difference between the winning and losing probabilities is 0.0526, which also represents the house advantage of having two zeroes (since 2/38 = 0.0526).

Suppose that the player now makes a compound bet on red and odd at the same time. It follows:

The probability of winning is 10/38 = 0.2632.

The probability of losing is 12/38 = 0.3158.

The probability of ending as draw is 16/38 = 0.4210.

The same probabilities are also true for betting on black and even at the same time. Again the difference between the probabilities of winning and losing is 0.0526. We consider such a compound bet is a less aggressive play while compared with a normal play since it generates a draw case. The chance of being a tie game is 0.4210. In other words, there is about 42% of the time the player will end up as draw.

Next, suppose that the player makes a compound bet on red and even at the same time. It follows:

The probability of winning is 8/38 = 0.2105.

The probability of losing is 10/38 = 0.2632.

The probability of ending as draw is 20/38 = 0.5263.

The same probabilities are also true for betting on black and odd at the same time. The difference between the probabilities of winning and losing is again 0.0526. We consider such a compound bet a least aggressive play while compared with a normal play. The chance of being a tie game is 0.5263. In other words, there is about 52% of the time the player will end up as draw.

For an European roulette which is the single-zero case, the probabilities can be computed the same way. Suppose that the player makes a simple bet on red, black, odd, even, small, or large numbers. It follows:

The probability of winning is 18/37 = 0.4865.

The probability of losing is 19/37 = 0.5135.

The difference between the winning and losing probabilities is 0.0270, which also represents the house advantage due to having a single-zero since 1/37 = 0.0270.

Suppose that the player makes a compound bet on red and odd at the same time. It follows:

The probability of winning is 10/37 = 0.2703.

The probability of losing is 11/37 = 0.5135.

The probability of ending as draw is 16/37 = 0.4324.

The same probabilities are also true for betting on black and even at the same time. The difference between the probabilities of winning and losing is again 0.0270. We consider such a compound bet a less aggressive play while compared with a normal play since it generates a draw case. The chance of being a tie game is 0.4324. In other words, there is about 43% of the time the player will end up as draw.

Next, suppose that the player makes a compound bet on red and even at the same time. It follows:

The probability of winning is 8/37 = 0.2162.

The probability of losing is 9/37 = 0.2432.

The probability of ending as draw is 20/37 = 0.5405.

The same probabilities are also true for betting on black and odd at the same time. The difference between the probabilities of winning and losing is again 0.0270. We consider such a compound bet a least aggressive play while compared with a normal play. The chance of being a tie game is 0.5405. In other words, there is about 54% of the time the player will end up as draw.

The question follow is whether it is possible to eliminate this differential. With the current 36-number scheme (0 and 00 uncounted for) it is not possible. Only within a 48-number scheme, it can be done. Simply add one more set of 12 numbers (from 37 to 48), then apply same double-looping technique introduced in Section 2 previously, it is no need to repeat it here. In other words, we shall have 12 each of red-odd, red-even, black-odd, and black-even numbers. Of course, all the odds, payoffs and layouts have to be adjusted accordingly. The new modified 48-number roulette may not be practically usable, since it further reduces the winning chance on a single-number bet which is already hard enough under the current scheme. However, it is the only alternative or possible trade-off.

5. A "Success-region" Method

In this method, we view the roulette as a spinning wheel experiment. The wheel is to be scaled from 0.00 to 2.00 uniformly. For each space or equal interval, the probability for the spin to stop is the same. The approach has been first mentioned by Fraser.[1] The theory behind such an experiment is uniform distribution. For an uniform probability distribution, its probability density function is:

$$f(x) = \begin{cases} 1(b-a) & a < x < b \\ 0 & \text{elsewhere} \end{cases}$$

and its distribution function is $F(x) = \int_{-\infty}^{\infty} f(x)dx$, or

$$F(x) = \begin{cases} (x-a)/(b-a) & a \le x \le b \\ 0 & x < a \\ 1 & x > b \end{cases}$$

As we apply it to case of roulette, we shall let a-value = 0, and b-value = 38. Since we have 38 spaces, or equal intervals on our wheel, and the probability for the spinner stopping on each space is 1/38 =0.0263.

There is a good reason to apply uniform distribution here. After the non-randomness had been proved, we may consider that the numbers on a roulette wheel are no longer a set of random numbers, but a sequence of natural numbers. Therefore, the success-region technique means to select an adequate segment of the sequence.

Mathematically, it can be stated as follows:

Let A_i be the probability of choosing ith number for our player, then $P(A_i) = 0.0263$, with $i = 1,\ldots,38$ and $\sum_{i=1}^{n} P(A_i) = P(A_1) + P(A_2) + \ldots + P(A_n) = 1.0000$.

Then, a player will choose his success-region as

$$P(A_1 \cup A_2 \cup \ldots \cup A_k) = \sum_{i=1}^{k} P(A_i), \text{ where } 1 \leq k \leq n.$$

and his failure-region will be:

$$P(A_{k+1} \cup A_{K+2} \cup \ldots \cup A_n) = \sum_{i=k+1}^{n} P(A_i), \ 1 \leq K+1 \leq n.$$

The key words in this method are segment or partial sequence. In other words, we are choosing several numbers in an entire sequence to cover the winning number as possible; through the use of uniform distribution.

Besides, in the process of making an American roulette, we found that for a three-way division, the probabilities for each region are not equal, two regions have higher probabilities than the other one. Cross reference with Figure 2 in Section 2 will be helpful. This finding can also support our theory.

6. Random walk or Markovian Model

In this section, we shall discuss the classical gamblers' ruin problem, which presents a typical random walk or Markovian model. It can be formulated as follows: A gambler has k units capital, and his opponent has (a-k) units capital. The total capital in the game is a, since $k + (a-k) = a$. The probability of his winning is p, and the probability of his losing is q, also $p + q = 1$. For each win, the gambler receives one unit of capital from his opponent, and for each loss, he pays one unit of capital to his opponent. Apparently, the gambler is taking a random walk along the capital axis. If he reaches the left end which means his ruin. If he reaches the right e d which means his success (gaining all the capital).

Let us concentrate only on the probability of his ruin; given all the a, k, p, q values, and let q_k indicate the probability of his eventual ruin, all we have to do is to solve a first order difference equation

$$q_k = p \cdot q_{k+1} + q \cdot q_{k-1}$$

with boundary condition $q_0 = 1$, and $q_a = 0$.

Its solution had been offered by many statisticians and mathematicians as follows:

$$q_k = \frac{(q/p)^a - (q/p)^k}{(q/p)^a - 1}.$$

An Algebraic solution is also provided by Kemeny, Snell, and Thompson.[2] By substituting $p/q = r$, $r<1$, then $q/p = 1/r$. The answer is

$$q_k = \frac{1 - r^{a-k}}{1 - r^a}$$

The discussion of "classical gamblers' ruin" problem is necessary here since it applies to all games of chance. My point is that since the number-arrangement on roulette wheels had been proved not random, "Red-odd versus black-even" differential had been revealed, and it is also found that the probabilities for some areas are not equal, these facts indicate that roulette is not flawless, it is not a perfect game of chance. These facts could offset a perfect game of chance. These facts could offset some of house advantages. In some cases, one play can have advantages over the other, for example, playing red and odd against playing red and even, etc. In other words, after non-randomness had been proved, roulette may no longer belong to random walk or Markovian models; therefore, the "classical gamblers' ruin" may not apply to the case of roulette.

7. Conclusion

Roulette is an interesting and intriguing game. It has perplexed people for quite some time. It is about time for roulette to be solved completely. In this paper, I had revealed some of its secrets, if not all. I have a few conclusions:

One of my main purposes is to make my colleagues aware that not to use roulette to be an example of randomness in writing statistical texts. This is a common error, some have done so. I have no difficulty to provide you a list of book titles, or you can find by yourself in the library or on your bookshelf. Even Von Neumann and Morgenstern might have overlooked on this in their classical work.[3]

Since I am a statistician, not a gambler, the statistical proof is my top priority, winning comes second. This does not mean that statisticians can not win. In this paper, I had clearly solved the structural or hardware part problem of roulette. To guarantee the win, one needs to do some more homework which means the software part work. I believe that both classical analysis and Bayesian analysis can produce winning programs. In other words, there may be only one hardware solution, but there are more than one software solution. Another analogy is that just like building an economic model, the structural equation is given, but coefficients still need to be filled in.

In this paper, I had mentioned computer before, but I did not use a computer. Anyone who had fundamental statistical training can tell that all I had applied is statistical reasoning. It is rather a manual or human job, not a computer job. Roulette is a typical problem for statisticians, not for mathematicians or computer programmers; for operational researchers, maybe. Of course, I shall not underestimate the power of a computer; it will be very useful in our next stage -- to write winning programs as I had mentioned earlier.

There is one conclusion on the theoretical aspect. After the non-randomness has been proved, Roulette may not belong to the family of random walk or Markovian models. Therefore, the "classical gamblers' ruin" may not apply to the case of roulette. It further leads to the conclusion that roulette is not a game of chance. More research work can be done along this general direction.

FOOTNOTES

*Thanks to Professor J. Stuart Hunter, our President, for approval of the topic, so it can be presented in the Section of Sports.

**There was an article in JASA (September, 1982) mentioned that roulette is a statistical problem. Since it is also a gambling in nature, I did copyright my basic solutions in 1982.

1. Fraser, (1960, pp. 65-67).

2. Kemeny et al., (1966, pp. 210-212).

3. Von Neumann and Morgenstern, (1947, p. 87).

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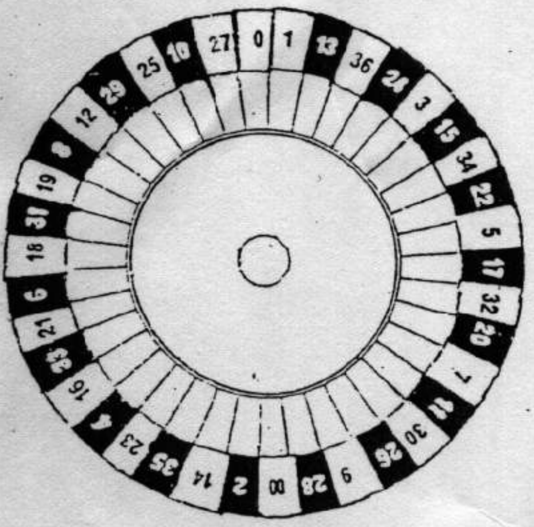

Figure 1.

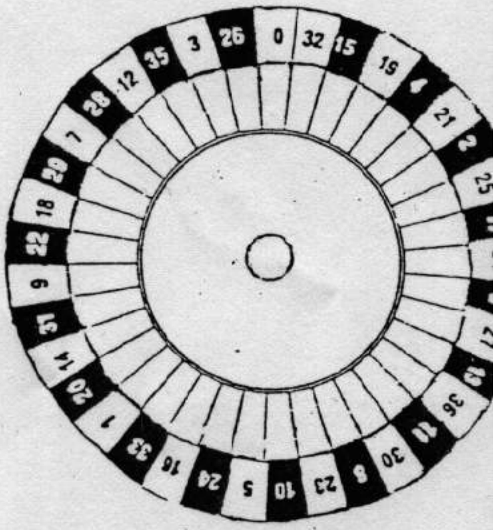

Figure 5.

Figure 10.

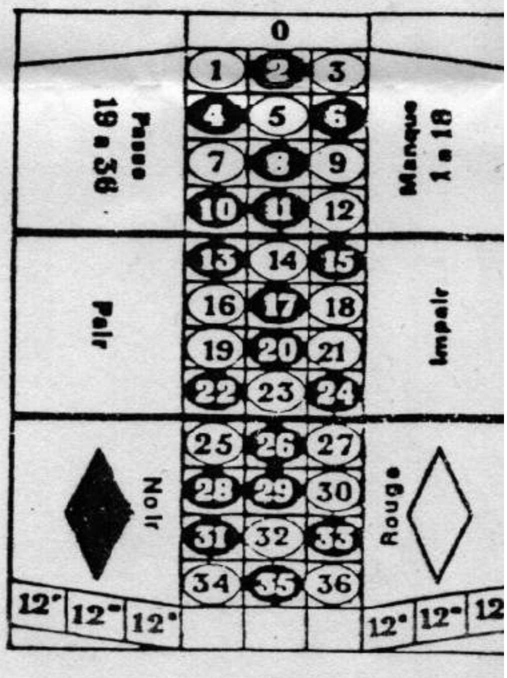

Figure 11.

0	
00	

1	3	5	7	9	11
2	4	6	8	10	12

13	15	17	19	21	23
14	16	18	20	22	24

25	27	29	31	33	35
26	28	30	32	34	36

Figure 2.

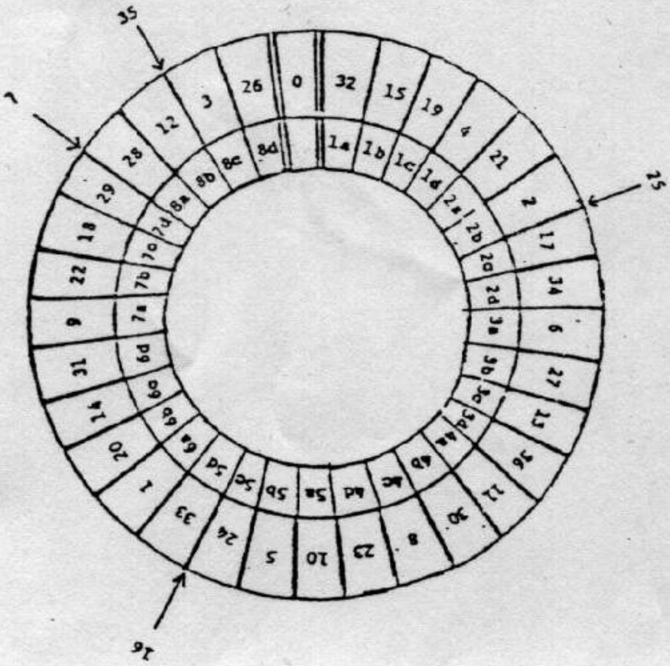

Figure 9.

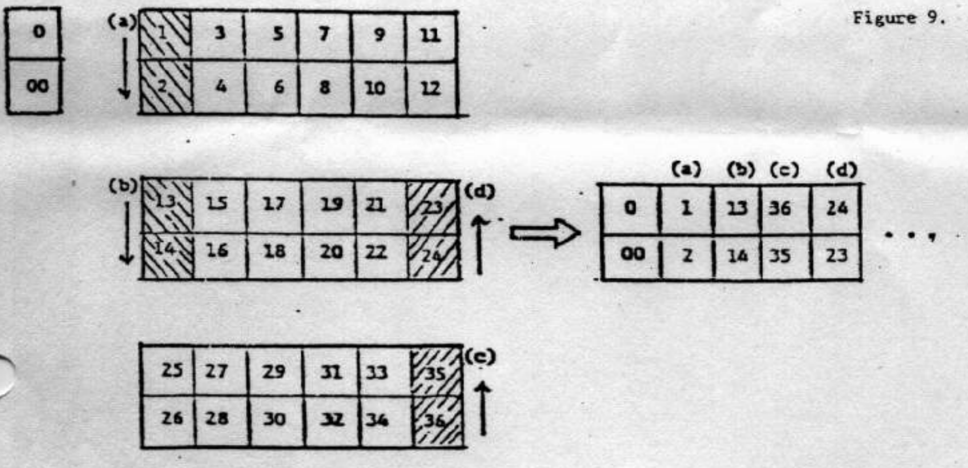

Figure 3.

0	1	13	36	24	3	15	34	22	5	17	32	20	7	11	30	26	9	28
00	2	14	35	23	4	16	33	21	6	18	31	19	8	12	29	25	10	27

Figure 4.

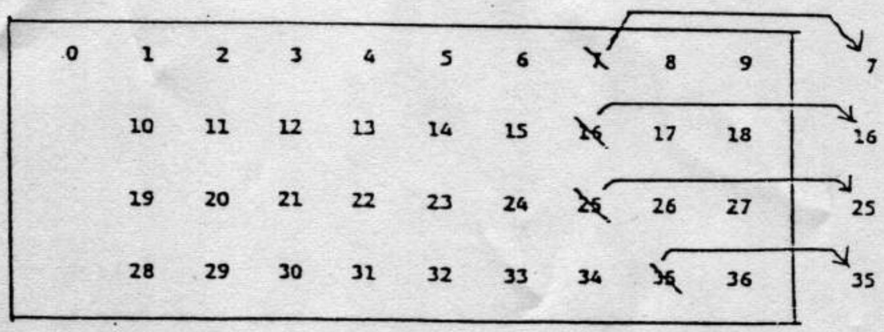

Figure 6.

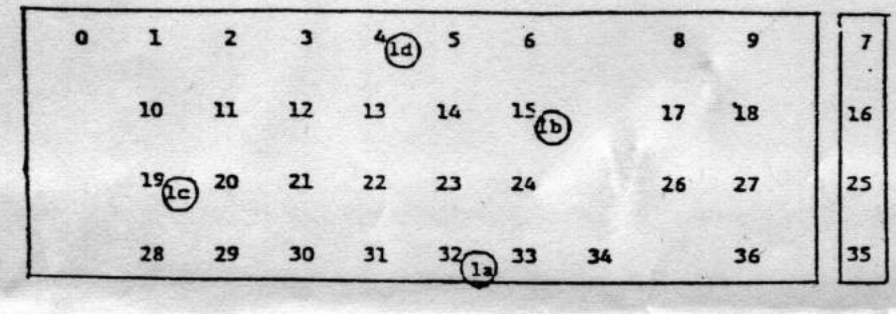

Figure 7.

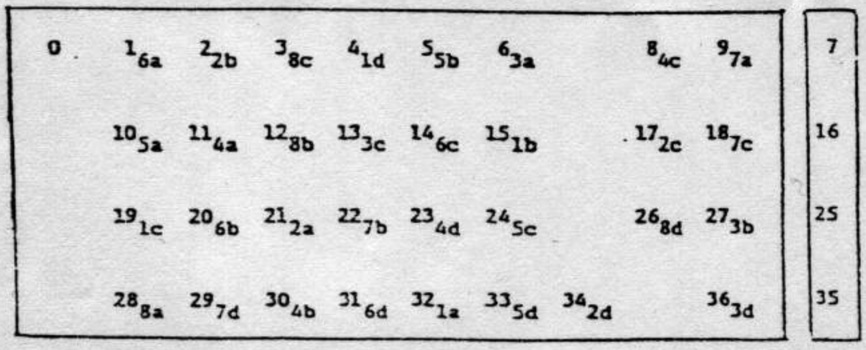

Figure 8.