THE LAW OF THE THIRD

A FUNDAMENTAL RESEARCH DISCOVERING THE MATHEMATICAL EQUATION RETULATING RANDOMNESS WITH REPLACEMENT

By

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Abstract

Every **Random** numerical activity **With Replacement** such as Roulette and Dice Games in the gaming industry, Lotteries etc. which have multiple **equally probable outcomes** that could occur is believed to be **Mutually Exclusive** and **Independent** that two outcomes cannot occur simultaneously and the next outcome does not depend on what occurred in the past.

This **Fundamental Research** clearly reveals the fact that such activities are regulated by a Generic **Linear Mathematical Equation**. Depending on the number of equally likely outcomes (N) that could occur, the nature approximately hides one third of N.

The Roulette Players and Casinos have visually observed this phenomenon for decades and termed it as the **Law of the Third**. In the event of European Roulette, there are 37 equally likely outcomes that could occur and it had been observed that within any 37 consecutive outcomes, there are only 23-24 distinct numbers (DNs) and the other 13-14 numbers are not present. In other words there are 13-14 numbers among the 37 that have repeated.

The **First Phase** of this research firmly established the fact that this observation made by the Roulette Players is valid by analyzing 30 data sets comprising 37 consecutive spins each, randomly obtained from the Live Spinning Mode in a Real Casino, Live Spinning Mode in an Internet Casino, Auto Spin Mode in an Internet Casino and Random Numbers Generated by the computer in an Internet Casino. All four modes firmly established compliance with the Law of the Third.

Based on such observation, the **Second Phase** of the research was commenced. The individual numbers from 1 to 50 were tested; each individual number 30 times by drawing them from a bin with replacement and the average number of DNs across the 30 samples were tabulated. Through a **Regression Analysis**, the **Best Fit Line** was established and the **Liner Mathematical Equation Y = 0.6291X + 0.2402** was derived. When X was equated to 37 for the data samples used in the first phase, Y became a value between 23-24, which proved the fact that the Law of the Third is not specific to Roulette but it is **generic** to any random numerical activity with replacement. The Y value therefore is termed as **Colonne's Value**.

The **Third Phase** of the research analyzed 200 consecutive draws of the most popular lottery in Sri Lanka which draws an alpha-numeric number from a machine comprising 7 compartments, one character of the alphabet except the letter "I" and a six-digit number (e.g. E225175), each letter drawn from a compartment containing 25 letters and each number drawn from six different compartments containing the individual numbers 0-9. The analysis revealed that the alphabetic character (1/25 probability), the first two numbers, the middle two numbers and the last two numbers (1/100 probability), are in strict compliance with the Law of The Third.

In summary, the probability of occurrence of a particular number as the next outcome is not 1/N and it varies and depends on whether it had already occurred within the immediate past outcomes equal to the Colonne's Value (23-24 in the event of Roulette). Thus, the **Old Hypothesis** of **Independent Events** needs to **be replaced** with the **New Hypothesis** of **Dependent Events** based on immediate Past Outcomes.

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Chapter One: Introduction

1.0 BACKGROUND

Albert Einstein is alleged to have said that the Roulette Table can be outperformed only by stealing money when the dealers are not looking. In other words, Roulette is a game in which perfect randomness prevails. On the contrary, Edward Lorenz in 1960, in his **Theory of Chaos**, observed that occurrences of a repeated activity may appear to be random and unrelated, but eventually a pattern emerges in the short term. Nassim Nicholas Taleb in his two books "Fooled By Randomness" and "The Black Swan" argues that such patterns associated with random events, observed either visually or mathematically, are mere illusions created by nature to fool the mind and nobody can outperform randomness.

At the outset, this research was commenced to explore the possibility of empirically validating the observation made by Edward Lorenz. In the process, three regulatory binary patterns were discovered and observed as to how such patterns topologically interact with each other and create a mega pattern in the form of a Sine Curve. An Application Software had been developed and offered free of charge to the world through the website www.neworiginalthinking.com to explore the trends, in regard to outcomes in a Roulette Table and make gains in the long run. Further, the operational content of this book is available in the free e-book "**DNA Of Roulette: Topological Interaction Of Three Mathematically Regulated Binary Patterns**", being made available in the same website. Four data sets comprising 30 data samples, containing 37 consecutive spins in each data sample, obtained by randomly entering ongoing sessions in a Real Casino on real-play mode and by randomly accessing a highly reliable Live Internet Casino on live-spin, auto-spin and computer-simulated (RNG) modes respectively, were used in this research.

The Roulette Players and Casinos for decades have observed that among any 37[1] consecutive spin outcomes, there are only 23-24 distinct numbers and 13-14 numbers are missing. In other words there are 13-14 numbers repeat within any 37 consecutive numbers. The roulette players have termed this phenomenon as the **Law of the Third.**

1.1 RESEARCH QUESTIONS

    1.1.1   Are the four sets of data stated above consistent with the observation made by the Roulette Players which is named by them as the Law of the Third?

    1.1.2   Is such observation specific to Roulette or is it a generic Law of Nature?

    1.1.3   If it is generic, are there any other applications other than Roulette which are in strict compliance with the Law of the Third?

1.2 JUSTIFICATION

If the research questions can be empirically validated and established, the **Old Hypothesis** of **Independent Events** needs to **be replaced** with a **New Hypothesis** of **Dependent Events** based on immediate past outcomes**.** Upon academic acceptance of this research, the fundamental assumption of the events being independent concerning randomness with replacement in the current Theories of Probability needs to be discarded and replaced with new theories. In other words, all the established theories of probability based on such assumption needs to be reexamined and rewritten.

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[1] European Roulette has 37 equally probable likely outcomes and American Roulette has 38 equally probable likely outcomes.

1.3 RESEARCH OBJECTIVES

The two prime objectives of the research are

    1.3.1   To empirically validate the Law of the Third
    1.3.2   To ascertain the mathematical equation underlying the Law of the Third.

1.4 SCOPE

    1.4.1   The research is confined to events that have less or equal to 100 equally probable likely outcomes. The reason being the impact of the value of the Intercept (c) of a Linear equation $Y = m.X + c$ becomes insignificant as the X value increases (m is the Gradient).

1.5 LIMITATIONS

    1.5.1   Since there is no other research being done on the Law of the Third prior to this research and it being a fundamental research, there are no references available to be cited in the literature survey.
    1.5.2   Research problem 1.1.1 is tested with European Roulette only.

Chapter Two: Literature Review

2.0 INTRODUCTION

Wikipedia provides the following explanation on random sampling and randomness with replacement.

Quote:
In statistics, a **simple random sample** is a subset of individuals (a sample) chosen from a larger set (a population). Each individual is chosen randomly and entirely by chance, such that each individual has the same probability of being chosen at any stage during the sampling process, and each subset of k individuals has the same probability of being chosen for the sample as any other subset of k individuals. This process and technique is known as **simple random sampling**. A simple random sample is an unbiased surveying technique.

Simple random sampling is a basic type of sampling, since it can be a component of other more complex sampling methods. The principle of simple random sampling is that every object has the same probability of being chosen. For example, suppose N college students want to get a ticket for a basketball game, but there are only $X < N$ tickets for them, so they decide to have a fair way to see who gets to go. Then, everybody is given a number in the range from 0 to N-1, and random numbers are generated, either electronically or from a table of random numbers. Numbers outside the range from 0 to N-1 are ignored, as are any numbers previously selected. The first X numbers would identify the lucky ticket winners.
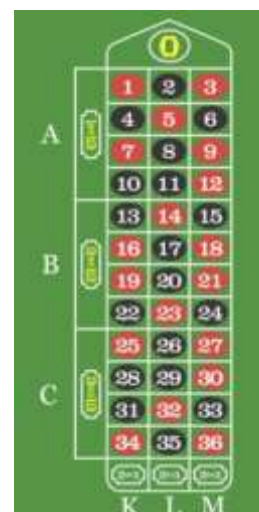
In small populations and often in large ones, such sampling is typically done "**without replacement**", i.e., one deliberately avoids choosing any member of the population more than once. Although simple random sampling can be conducted with replacement instead, this is less common and would normally be described more fully as simple random sampling **with replacement**. Sampling done without replacement is no longer independent, but still satisfies exchangeability, hence many results still hold. Further, for a small sample from a large population, sampling without replacement is approximately the same as sampling with replacement, since the odds of choosing the same individual twice is low.
Unquote:

If there is no possibility of any two outcomes occurring together, the events are said to be Mutually Exclusive. E.g. If a coin is tossed, Heads or Tails cannot occur together. Similarly, if a dice is rolled, the outcome is just one out of the six likely numbers that could occur.

All the data samples used in this research are driven by the criteria "Mutually Exclusive" and "Independent".

2.1 ROULETTE TABLE

In a European Roulette Wheel, there are 37 equal sized slots forming a circle to represent numbers 0 to 36. The numbers do not follow any sequence, but arranged according to a global convention. Out of the numbers 1 to 36, 1-18 are termed as LOW and 19-36 are termed as HIGH. Also there are 18 ODD numbers and 18 EVEN numbers and 18 RED numbers and 18 BLACK numbers and 0 is GREEN. The only order which is observed on the roulette wheel is that the colours RED and BLACK are arranged in an alternating manner.

In Roulette Table Layout, the table is vertically and horizontally divided into three portions each. The vertical blocks are termed as DOZENS and the horizontal blocks are termed as COLUMN, containing 12 numbers each. Most importantly, it must be observed that there are asymmetries associated with two out of three individual DOZENS (one has only low numbers and one has only high numbers) and two out of three individual COLUMNS (one has eight blacks and four reds and one has four blacks and eight reds)[2], on the roulette table layout.

it can be clearly observed that the statistical balances are perfectly maintained among all wagering categories (HIGH/LOW, RED/BLACK, ODD/EVEN, DOZENS, COLUMNS and NUMBERS) in the long run. This clearly implies that perfect randomness prevails in the long run from all perspectives and the person(s) who spins the ball have no control over the outcomes. Also, it can be firmly established that the asymmetries associated with Dozens are more rigorous than the asymmetries associated with Columns.

A Ball is spun at a high speed either manually or mechanically along the edge of the wheel and upon gradually losing the speed and hitting many breakers inside, the ball falls into one of the equally sized slots.

2.2   MAHAJANA SAMPATHA LOTTERY

This most popular lottery is drawn thrice a week in Sri Lanka alpha-numeric number from a machine comprising 7 compartments is drawn, one character of the alphabet except the letter "I" and a six-digit number, each letter drawn from a compartment containing 25 letters and each number drawn from six different compartments containing the individual numbers 0-9.

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[2] Some roulette tables do not have column asymmetries and SYSTEM RCLV has not been tested for such tables.

Chapter Three: Research Design

3.0   RESEARCH METHODOLOGY

3.0.1   In order to test the research problem 1.1.1, the same data sets used for the pattern observations in the book titled "**DNA Of Roulette: Topological Interaction Of Three Mathematically Regulated Binary Patterns**", authored by the researcher and published in December 2009.

3.0.2   For the purpose of establishing the mathematical equation underlying the Law of the Third as per the research problem 1.1.2, numbers from 1 to 50 were used from a Bingo number set; each number is tested 30 times by putting the corresponding number into a bin, shaking it, drawing numbers with replacement and tabulating the distinct numbers (DNs) present. The average of DNs across the 30 samples was used in the Regression Analysis to determine the Best Fit Line.

3.0.3   The winning numbers of 200 consecutive Mahajana Sampatha Lottery draws (2382 to 2581) are used for the analysis.

In order to empirically validate the three following hypothesis, the independent variable X is assumed to be all equally probable likely outcomes, which are mutually exclusive and independent (inferring randomness with replacement) and the dependant variable Y is assumed to be the number of distinct numbers present among X successive outcomes.

3.1 HYPOTHESES

3.1.1   **HYPOTHESIS 1**

H1: The average number of distinct numbers present within any 37 consecutive numbers in European Roulette is around 23-24, irrespective of the mode of spinning.

3.1.2   **HYPOTHESIS 2**

H2: The Law the Third is a generic rule of nature represented by a Linear Mathematical Equation and it regulates all activities involving numbers subject to randomness with replacement.

3.1.3   **HYPOTHESIS 3**

H3: There are other applications of the Law of the Third.

Chapter Four: Data Analysis

4.0   DATA ANALYSIS

A hypothesis could be described as a proposition or a specific statement of predication, the validity of which remains to be tested. The hypothesis that supports the predication is called the Alternate Hypothesis. The hypothesis that describes all the remaining possible outcomes is called the null hypothesis.

HYPOTHESIS 1

H1: The average number of distinct numbers present within any 37 consecutive numbers in European Roulette is around 23-24, irrespective of the mode of spinning and approximately 1/3 of the likely numbers have not appeared within 37 consecutive spins, a phenomenon termed as the Law of the Third by the Roulette players.

The analysis of data tables specified under 2.0.1 reveals that the average occurrence of Distinct Numbers within 37 consecutive spins is 23-24[3] per 37 consecutives spin outcomes and it is highly consistent among individual data tables, irrespective of the mode of spinning.

Table 1: Live Spin / Real Casino

AVERAGE DISTINCT NUMBERS	DOZENS			COLUMNS			HIGH/LOW		RED/BLACK		ODD/EVEN	
N/37	A	B	C	K	L	M	H	L	R	B	O	E
22.97	11.33	12.43	12.17	11.43	11.90	12.60	18.13	17.80	17.90	18.03	19.27	16.67

Table 2: Live Spin / Internet Casino

AVERAGE DISTINCT NUMBERS	DOZENS			COLUMNS			HIGH/LOW		RED/BLACK		ODD/EVEN	
N/37	A	B	C	K	L	M	H	L	R	B	O	E
23.73	12.13	11.93	11.93	12.43	10.70	12.87	18.00	18.00	17.83	18.17	17.27	18.73

Table 3: Auto Spin / Internet Casino

AVERAGE DISTINCT NUMBERS	DOZENS			COLUMNS			HIGH/LOW		RED/BLACK		ODD/EVEN	
N/37	A	B	C	K	L	M	H	L	R	B	O	E
24.00	11.80	12.07	12.23	11.77	11.67	12.67	18.53	17.57	18.20	17.90	18.53	17.57

Table 4: Computer Simulated Random Numbers / Internet Casino

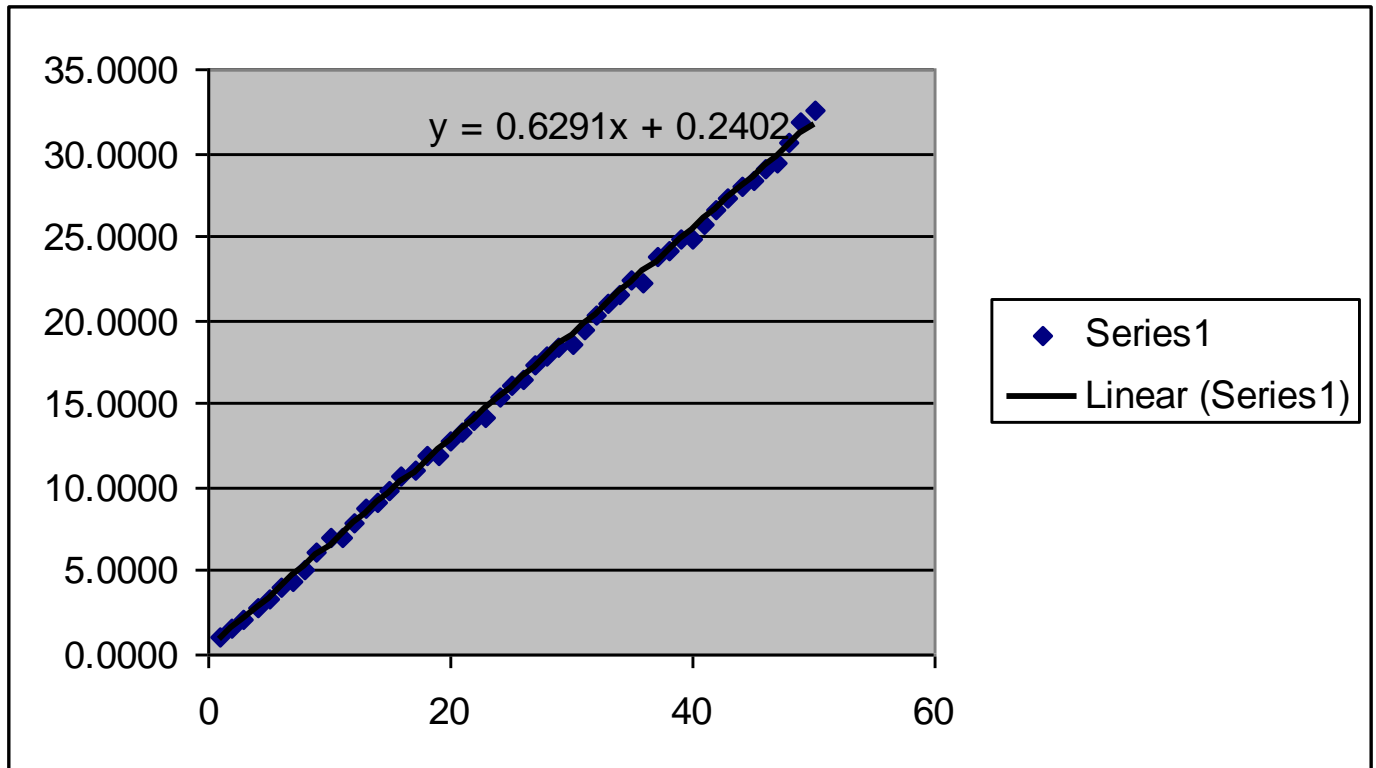| AVERAGE DISTINCT NUMBERS	DOZENS			COLUMNS			HIGH/LOW		RED/BLACK		ODD/EVEN	
N/37	A	B	C	K	L	M	H	L	R	B	O	E
23.60	11.63	12.80	11.27	11.77	11.33	12.60	17.63	18.07	18.27	17.43	17.67	18.03

It is clearly evident that all data sets are in conformity with the mathematically determinable probabilities in the long run, subject to the Law of the Third. Therefore, the **Hypothesis H1** can be accepted as **TRUE**.

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[3] Roulette Players those who have observed this inexplicable phenomenon call it the "Law of the Third".

HYPOTHESIS 2

H2: The Law the Third is a generic rule of nature represented by a Linear Mathematical Equation and it regulates all activities involving numbers subject to randomness with replacement.

For Regression purposes, each X number was tested for 30 data samples (from X = 1 to X = 50) and the mean value of distinct numbers in the 30 data samples was assumed to be the Y value corresponding to X. The analysis of the Data Table 5 clearly establishes the fact that the Law of the Third is a generic Rule of Nature depicted by the Liner Mathematical Equation **Y = 0.6291X + 0.2402**.

If numbers are drawn X times from a collection **of** X different numbers with replacement, **Y = 0.6291X + 0.2402** distinct numbers will be present among the X numbers drawn. The Whole Number[4] pertaining to Y value is termed as **COLONNE'S VALUE**. Thus, the **Hypothesis H2** can be considered as **TRUE**.

Also, another significant observation can be made that when X is equated to 37, the Y value becomes 23.51, which perfectly **rationalizes** the validity of **Hypothesis H1**.

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[4] Rounded up to the nearest integer.

HYPOTHESIS 3

> H3: There are other applications of the Law of the Third.

The analysis of the Data Table 6 clearly establishes the fact that there are other activities that are regulated by the Law of the Third, by the Liner Mathematical Equation **Y = 0.6291X + 0.2402**.

The Columns A, B & C represent the first two, middle two and the last two numbers in the 200 consecutive draws of the Mahajana Sampatha Lottery (100 equally probable likely outcomes). The Column D represents the Alpha character (25 equally probable likely outcomes). The first line provides the average number of distinct numbers present as empirically observed in the data sample and the second line gives the Colonne' Values derived from the mathematical equation corresponding to X = 100 and X = 25, respectively.

A	B	C	D
60.178	62.465	62.198	15.665
63.150	63.150	63.150	15.968

Thus, the **Hypothesis H3** can be considered as **TRUE**. Further, it supplements the validity of the Hypotheses **H1** and **H2.**

Chapter Five: Discussion

5.0 **OVERVIEW**

The "**Law of the Third**" as observed and named by the Roulette Players is that approximately 1/3 of X (X = 37 for European Roulette and X = 38 for American Roulette) different numbers do not appear within X consecutive outcomes associated with a repeated random activity in quick succession, with replacement. After an extensive and a rigorous testing, the generic equation underlying this observation had been empirically established that if numbers are drawn X times from X different numbers with replacement, only **Y = 0.6291X + 0.2402** distinct numbers will be present among the X numbers drawn. Numbers ranging from 01 to 50 have been tested in the process of deriving the above generic equation. Each number had been tested 30 times with replacement and the average of distinct numbers present over the 30 data samples corresponding to each number (ranging from 01 to 50) were used as inputs in a regression analysis to establish the above equation. The best-estimated rounded-up Y value is termed as **Colonne's Value**.

When 30 data samples comprising 37 consecutive spins in each sample taken from European Roulette tables in real and internet casinos using all three modes of spinning such as live, auto and computer-simulated were analyzed, majority of the data samples had 24 distinct numbers and the average number of distinct numbers per data sample was also found to be 24. If X is equated to 37 in the above equation, the rounded up whole number pertaining to Y value becomes 24.

With the discovery of the mathematical equation of the Law of the Third, the conventional wisdom in regard to randomness with replacement pertaining to equally probable likely outcomes perceived to be independent holds only until the activity is repeated Y times with replacement in quick succession. The moment the availability of past records becomes greater or equal to the Colonne's Value (Y), the probability of the next number to occur becomes dependent on the number of distinct numbers present (assumed to be N) within the Y number of past records.

5.1 **CONTRADICTORY & CONFRONTATIONAL INFERENCES TO THE EXISTING KNOWLEDGE**

Upon reaching the Colonne's Value in terms of repeated equally likely consecutive outcomes:

5.1.1 Within the next X − Y forthcoming consecutive events to be repeated in quick succession, the probability of occurrence of a number which does not belong to the N distinct numbers within the past Y consecutive spins is (Y − N) / (X − N) in contrast to the existing knowledge ((X − Y) / X).

For Example, if X = 37, Y = 24 and N = 20, only 4 (= Y − N) out of 17 (= X − N) numbers which are not among the 20 (N) distinct numbers are likely to occur within the next 13 (= X − Y) consecutive events. Under the current assumption of the events being independent, such probability would have been 13/37 instead of 4/17.

5.1.2 In view of 5.1.1, the probability of a number among the N distinct numbers within the Y past outcomes occurring within the next X − Y forthcoming consecutive events is (X − 2Y + N) / N, in contrast to the (X − Y) / X probability under the existing knowledge.

Using the same example in 5.1.1, the probability of occurrence of a number among the 20 numbers that have already occurred within the next 13 consecutive spins is 9/20 in contrast to the 13/37 probability under the current assumption.

5.1.3 In view of 5.1.1 & 5.1.2, it is implied that that after reaching the availability of Y past outcomes, the probability of occurrence of a number which is not included in N distinct numbers as the next spin outcome is (Y − N) / ((X − N)(X − Y)) and the probability of occurrence of a number which is among the N distinct numbers as the next spin outcome event is (X − 2Y + N) /((N(X − Y)), against the conventional wisdom of 1/X.

Also, such probabilities could vary from event to event as the N value can change from event to event. Also, such probabilities could vary from spin to spin as the N value can change from event to event.

Using the same example in 5.1.1, the probability of occurrence of a number which is not included in 20 distinct numbers as the next spin outcome is 4/231 (< 1/37) and the probability of occurrence of a number which is among the 20 distinct numbers as the next spin outcome event is 9/260 (> 1/37), against the conventional wisdom of 1/37.

Chapter Six: Conclusion & Recommendations

5.0 **CONCLUSION**

The precision accuracy of the Gradient (m = 0.6291) and the Interception (c = 0.2402) is not significant in regard to making decisions based on the Law of the Third, as only positive integers are used as Y value for such purposes in reality and the variation could only be either +1 or -1, in integer terms. As the X value becomes higher, the impact arising from the variation becomes further insignificant to make decisions. Also, there always can be marginal deviations and exceptions that could occur in regard to the probabilities defined above.

As per the empirical citations elaborated above, a fact is clearly evident that any alpha-numerical activity upon reaching the respective Colonne's Value, is being regulated by the nature using a simple linear equation. From there onwards, the predictability increases. Also, another assumption can be made that if the nature is regulating such outcomes, it is done by creating patterns, as stated by Edward Lorenz in 1960. The researcher has already discovered three regulatory binary patterns associated with the Dozens and the Columns of the Roulette Layout and perfected a computer program by optimizing on the topological interaction of such patterns, which is given free of charge to the world through is website www.neworiginalthinking.com.

Most importantly, it can be clearly established that the **Old Hypothesis** of **Independent Events** assumed in the conventional probability theories needs to **be replaced** with the **New Hypothesis** of **Dependent Events** based on Past Outcomes**.**

5.1 **RECOMMENDATIONS**

From a practical perspective, the concept of Arc Elasticity of Demand in Micro Economics can be emulated in the new Paradigm of randomness with replacement as an **Arc Probability** coming into effect pertaining to the (X − Y) forthcoming events, provided that a minimum of Y past records are available. Also, the equation underlying the Law of the Third being generic, it can be emulated into many other areas such as dice games and lotteries.

Also, further research may be undertaken on other applications such as Dice Games, Analysis of the number plates of Vehicles etc. to further establish the Law of the Third.

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